Gyroscopes and Cyclones.

BY F. J. B. CORDEIRO.

The most familiar form of gyroscope is the top. The motion of such a body, spinning about an axis of symmetry and actuated by no other force except that of gravity, is easily derived from equations expressing the conservation of the living forces and the conservation of the momental areas about a vertical axis passing through the point of support.

Two equations can thus be obtained by which the angle ϑ—the inclination of the axis to the vertical—and the angle ψ—the azimuth of the plane passing through the axis and the fixed vertical line—can be found by quadratures. These equations were first obtained by Lagrange in his Mecanique Analytique, and were afterwards given by Poisson in his Traite de Mecanique. The former passes them over with but slight notice and proceeds to discuss the small oscillations of a body of any form suspended under the action of gravity from a fixed point. The latter limits the equations to the case in which the body has an initial angular velocity only about its axis, and applies them directly to determine the small oscillations of a top (1) when its axis is nearly vertical, and (2) when its axis makes a nearly constant angle with the vertical. After these necessarily limited results no effort seems to have been made for many years to get a clear conception of the nature of the forces at work. In the preface of Barnard's "Rotary Motion" we find "For some time the impression prevailed in the popular mind that the phenomena exhibited by the apparatus (gyroscope) could not be explained by natural laws. This idea was perhaps strengthened by the name applied to it by Prof. Olmstead (of Yale College), who called it "The Mechanical Paradox."

Quoting further from the same author, "After reading most of the popular explanations of the above phenomena given in our scientific and other publications, I have found none altogether satisfactory. While with more or less success, they expose the more obvious features of the phenomenon and find in the

force of gravity an efficient cause of horizontal motion, they usually end in destroying the foundation on which their theory is built, and leave an effect to exist without a cause: a horizontal motion of the revolving disc about a point of support is supposed to be accounted for, while the descending motion, which is the first and direct effect of gravity (without which no horizontal motion can take place) is ignored or supposed to be entirely eliminated. Indeed, it is gravely stated as a distinguishing peculiarity of rotary motion that, while gravity acting upon a non-rotating body causes it to descend vertically, the same force acting upon a rotary body causes it to move horizontally. A tendency to descend is supposed to produce the effect of an actual descent; as if, in mechanics, a mere tendency to motion ever produced any effect whatever without that motion actually taking place."

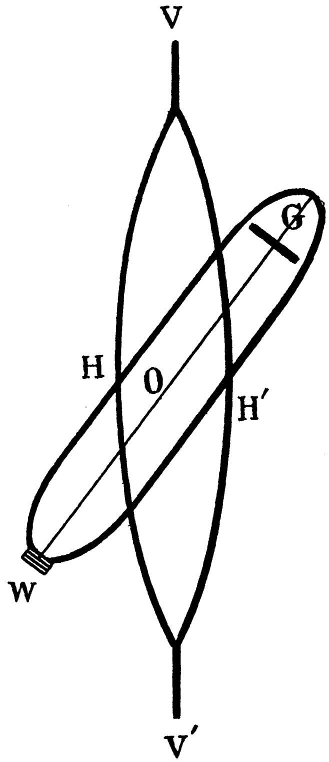

FIG. 1.

It was Major (afterwards General) Barnard of the army engineers, who first translated the cryptic, oracular meaning of Poisson's equations into clear everyday English, so that the motions of a gyroscope became clear to the everyday intellect. This was in 1858.

In the rotation of a gyroscope about a fixed axis, centrifugal forces must be developed. This Barnard did not consider in his discussion of the problem. As in the case of a gyroscope with a very rapid rotational motion, the precessional motion is slow, this force will be very small and it can be neglected. But in the theory of the gyroscope in general it must be considered.

Let us consider a gyroscope of the form shown in Fig. 1.

The gyroscope G is counterbalanced by the weight W, so that the centre of gravity of the instrument always remains fixed at O. It can turn about a vertical axis VV', and a horizontal axis HH'. Under these circumstances whatever motions are imparted to it, it will not be influenced by gravity.

If an impulsive spin be given to the gyroscope setting up an angular velocity ω, and if it be given an impulsive turn about the axis VV', setting up an

angular velocity Ω, its subsequent motion will take place under the action of no external applied forces. Intrinsic forces will be set up, however, modifying its motion continually. If the wheel were not rotating and it were to turn about the axis VV', the centrifugal forces would shortly bring the axis GW to a horizontal position. But if the spin and the turn are both counter-clockwise, a deflective gyroscopic force is set up tending to bring the axis GW into coincidence with the axis VV'. If at the outset of the motion, the component of the centrifugal forces normal to the axis GW, which tends to depress the axis, and the gyroscopic force normal to the axis GW, which tends to raise it, are equal; since they are opposite, there will be no motion of the instrument about the axis HH'.

Let us imagine the equimomental spheroid (Poinsot's momental ellipsoid) at O, with its major axis coincident with the axis GW, rigidly attached to the instrument and rolling on a tangent plane perpendicular to the axis VV'. Such a body will revolve about a line OI connecting the fixed centre O with the point of contact of the spheroid with the tangent plane, as an instantaneous axis. Let the axis GW make an angle γ with the vertical fixed axis OV and an angle i with the moving instantaneous axis OI. Now if the instrument is moving with a uniform angular velocity Ω about the axis OV, and with a uniform angular velocity ω about the axis OG, it will be in the condition of a body moving under the action of no forces, to which an original impulsive couple about the axis OV has been imparted, and one point of which O is fixed. It is shown in treatises on dynamics that under these circumstances certain conditions must hold between the angles in question, γ, i, and the angular velocities about OV and OG. The motion of such a body is exactly represented by causing its momental spheroid to roll on the tangent plane, and the following condition must hold: $\tan\gamma = \frac{A}{C}\tan i$.

Consequently when our gyroscope revolves uniformly about the axis OV, its motion can be represented by causing its momental spheroid to roll upon a perpendicular tangent plane, while its centre is fixed at O.

But we have found another condition requisite for such motion. The centrifugal and gyroscopic forces must balance each other in their turning effort about the axis HH^1. Let us see if these conditions are identical. Drop a perpendicular GM from G to OV. The centrifugal force (exerted in the direction

MG) is $\Sigma mr\Omega^2$. Now the rotation of the gyroscope can clearly have no effect upon this force. Therefore the centrifugal force $= M \, . \, OG \, . \sin \gamma \Omega^2$.

The component of this force perpendicular to the axis OG is

$$M \, . \, OG \, . \sin \gamma \, . \cos \gamma \Omega^2.$$

The gyroscopic force (exerted at right angles to the axis OG) is

$$M \, . \, \frac{k^2\omega \, . \, \Omega \, . \, \sin \gamma}{OG}.$$

These forces are opposed and must be equal. Therefore $OG^2 \, . \cos \gamma \Omega = k^2 \omega$. But $\Omega \sin \gamma = \omega \tan i$ from the conditions of the motion, where in the above equations M is the mass and A and C are the principal moments about the axis OG. Therefore $\tan \gamma = \frac{A}{C} \tan i$; and we see that the reason that a body under the action of no forces moves about its invariable line at a constant angle is because the centrifugal and gyroscopic forces are in equilibrium about its axis.

Now if at the outset of motion, the angular velocities Ω and ω do not fulfil the condition $\Omega \cos \gamma = \frac{C}{A} \omega$, these forces will not be in equilibrium and the gyroscope will tend to raise or lower itself until such condition is satisfied. The excess of one force over the other will give it a decreasing angular acceleration until it reaches the position of equilibrium. But it will be carried beyond this by its inertia and an increasing acceleration in the opposite direction will bring it to rest and finally back through the position of equilibrium. It will thus oscillate continually through the position of equilibrium as a centre.

The above discussion is of importance in connection with the theory of cyclones. With the establishment of a cyclone and its invariable rotation in a counter-clockwise direction in the Northern Hemisphere, we have nothing to do here. That can be found elsewhere. (See "The Problem of the Cyclone," Monthly Weather Review, November, 1903.) We shall regard a cyclone dynamically as a rigid rotating body—in other words as a gyroscope. That this is permissible is evident from the fact that a stream of water (or air) when directed through a tube curled into the form of a spiral, so that there is a rotating mass, exhibits all the properties of a gyroscope, which is nothing more than rotating matter. We can therefore substitute for any given cyclone a dynamically equivalent gyroscope. For the properties of a gyroscope are derived eventually

from the inertia of matter, and whether solid, liquid or gaseous, all matter possesses this essential property of inertia. Now we have used above a coiled spiral tube simply to direct the stream lines, while in a cyclone no such artificial constraint is necessary, since the forces at work in the body of a cyclone preserve the stream lines in a constant relation to each other. We can resolve the motion of the inflowing and outflowing air into two components—one of motion to and from the centre—and the other of circular motion about the centre as an axis. For every component directly towards the centre there is a component away from the centre, so that they eliminate each other. We therefore simply substitute for the circular motion a dynamically equivalent gyroscope—all that is necessary being that the moment of inertia of the two should be equal. And in fact we shall find that the calculated motion of such a gyroscope corresponds with the actual motion found in cyclones.

Let us suppose that a cyclone exists at some point of the Earth's surface and that there are no frictional forces. In other words it is free to preserve a constant rotational energy and to move over the surface of the Earth without resistance. Let us suppose that the rotational and azimuthal angular velocities ω and Ω, are so adjusted that $\tan\gamma = \frac{A}{C}\tan i$, where γ is the angle the axis of the cyclone makes with the axis of the Earth.

In such a case there will be uniform azimuthal motion about the axis of the Earth and the cyclone will preserve a uniform latitude. If the velocity of the Earth at the point underlying the centre of the cyclone is equal to that of the cyclone, the cyclone will remain stationary with respect to the Earth.

If less or greater, then there will be a corresponding motion of the cyclone relatively to the surface of the Earth, but it will move along a parallel of latitude. If ω and Ω do not fulfil the above condition, since the cyclone originally started revolving about the axis of the Earth—*i. e.* the elements of which it is composed had a definite moment about this axis,—it will have to conserve the original moment of momentum about this axis. In order to obtain stable motion therefore, it will strive to put itself in such a position that the invariable angular velocity ω and the variable velocity Ω will assume such a relation that the condition $\tan\gamma = \frac{A}{C}\tan i$ holds, i being the inclination of the instantaneous axis to the axis of the cyclone which varies in the adjustment referred to. Since the

relation $\frac{A}{C}\cos\gamma = \frac{\omega}{\Omega}$ holds, if ω is too large relatively to Ω, the axis of the cyclone will move to the North until it adjusts itself; if too small, the axis will move to the South. Now in the great majority of all cyclones ω is too large relatively to Ω, and the general movement is to the Northward. Still there are a very few cyclones which move along a parallel of latitude—at least during the first part of their motion—(Piddington has charted such a cyclone—) and very

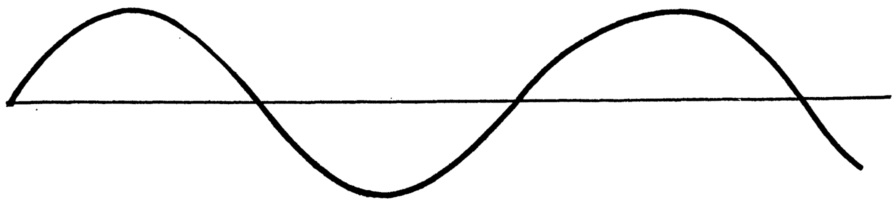

FIG. 2.

fewer still move to the Southward. It is evident that these latter can have very little energy and such we find to be actually the case. A very violent cyclone is apt to move rapidly to the North.

The position of equilibrium of most cyclones is apt to be in high latitudes. As we have seen before, in our experimental gyroscope, this position is apt to be overshot and a succession of oscillations backwards and forwards over this latitude generally occurs. If such an oscillation were developed upon a plane moving surface, the shape of the path would be a series of waves as in Fig. 2.

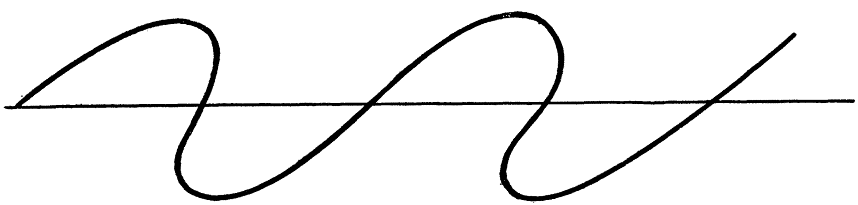

FIG. 3.

But the actual path of a cyclone is developed upon the moving surface of the Earth, and the velocity of the cyclone in higher latitudes is relatively much greater to the velocity of the Earth's surface than it is in lower latitudes. The path therefore will assume the form in Fig. 3.

This is a prediction from purely theoretical considerations. Now on examining the charted paths of cyclones crossing the Atlantic in high latitudes, we find many that exactly execute such a path. If a cyclone could preserve a

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constant rotational energy, after finding its latitude of equilibrium, it would continue forever circling around the Earth and oscillating about this latitude, in such a path as is pictured in Fig. 3. Abercrombie states that he has traced a cyclone which, starting in the Philippines, went up North to Japan, crossed successively the Pacific, the American continent and the Atlantic Ocean, and then passing over Russia, died out finally on the Steppes of Siberia, having thus almost completely circled the globe.

The writer has previously in a popular article (Bulletin of the American Geographical Society, No. 3, 1900) called attention, he believes for the first time, to the fact that the motion of cyclones is due chiefly to their own intrinsic forces, among which the gyroscopic forces developed are most important. In a later article ("The Problem of the Cyclone," Monthly Weather Review, November, 1903), he sketched out roughly a method of attacking the problem, taking into consideration the effect of frictional forces on the Earth's surface.

The previous explanations of the motions of cyclones, if they can be considered explanations, have been grossly inadequate. We shall mention here only two of these and afterwards consider carefully to what extent, howsoever slightly, they can modify a path which we have shown is due to other and greatly preponderating causes, viz.—the causes we have just discussed together with friction. The first of these explanations is that they are blown along with the prevailing winds. Tropical cyclones usually start out in the doldrums where the winds are few and uncertain. It is pointed out that the general circulation of the atmosphere has a westward component in lower latitudes and an eastward component in higher latitudes. These blow the cyclones first west then east, but how they work up to the northward is not made clear. It is hardly worth while to rebut by serious proof—which can be done—such an explanation.

The second is that cyclones are governed in their motions by the existence of some distant High or Low. The gradient may be almost imperceptible, but according to this theory, should a cyclone arise on a line between a High and a Low, no matter how far apart, it is considered that it ought to move towards the Low, *i. e.* after a fashion. But very often it does not do so, not even after a fashion.

We shall now attempt to analyze the motion of a cyclone taking into consideration the frictional forces developed. The writer had hoped that some one else with greater facilities and ability might have taken up the problem as indicated in the article mentioned above in the Monthly Weather Review, but the

possibility of predicting the path of a cyclone, if not for its complete run, certainly for a few days ahead, does not seem to have aroused any interest. There was a time when the electrician did not need to be a mathematician, but that day has passed. So with the study of Meteorology; the meteorologist of the future will have to be a mathematician first. In the cyclone, the grandest phenomenon by far, the Chef d'Oevre of his specialty, he will find a complicated problem of forces which can only be unravelled by the calculus of mathematics.

A cyclone rotating about its axis experiences a certain amount of friction as its under surface passes over the surface of the Earth. If the initial momental energy were not added to, it would soon come to rest by reason of this resisting couple. Now this friction couple exerts a force contrary to the original momental energy about the axis of the Earth and is therefore continually decreasing it. The original moment of momentum about the Earth's axis is expressed by the quantity $Mk^2 \omega \sin \vartheta + MR^2 \cos^2 \vartheta \,.\, \Omega$. where ϑ is the latitude of the cyclone. Since we shall suppose that by added energy the moment $Mk^2 \omega$ is kept always constant throughout the period that we are studying, it follows that the retarding frictional couple will be continually reducing the term $MR^2 \cos^2 \vartheta\, \Omega$. This at first will affect chiefly the velocity Ω, and the cyclone will tend to lag further and further behind the position it would have occupied if there were no friction. In a certain sense it can be considered to be continually screwing itself backwards. Now if the cyclone is urged backwards along a parallel of latitude this motion is opposed by frictional forces, so that the moment of the force opposing the original momental energy about the axis of the Earth, will be reduced by this amount. If l is the moment of the frictional force about the axis of the cyclone, which we suppose to be kept constant, and $2a$ is the arm of this couple and f is the force it exerts on this arm, and if f_1 is the force urging the cyclone backwards, then $l = f \,.\, 2a = f_1 \,.\, R \cos \vartheta$.

Now if f_{11} is the frictional resistance opposing the sliding of the cyclone bodily, we have for the effective backward moment $l_1 = l - f_{11} R \cos \vartheta$.

We can write the equation therefore

$$MR^2 \cos^2 \vartheta\, \Omega + Mk^2 \omega \sin \vartheta = K - M \int_0^t l_1\, dt \qquad (1)$$

where K is the moment of momentum about the Earth's axis at the beginning of the period under consideration.

Now the gyroscope will in general be urged away from its parallel of lati-

tude (in the great majority of cases, to the North, as already explained) and the normal effective deflecting force will be

$$\frac{Mk^2 \omega \,.\, \Omega \,.\, \cos \vartheta}{R} - MR \cos \vartheta \,.\, \sin \vartheta \,.\, \Omega^2 - Mf_{11} \,. \tag{2}$$

We can consider that the last two forces are in the nature of constraints and the remaining effective force is given by the expression (2) so that the body moves northward under this force without friction.

The total moment of momentum given in (1) will not be diminished therefore by this northward motion of the cyclone.* The term $Mk^2 \omega \sin \vartheta$ will be increased to be sure, but this will be made up by a loss in the first term. In other words the northward acceleration is at the expense of a certain amount of the momental energy of the cyclone about the Earth's axis.

We can write therefore

$$\frac{Mk^2 \omega \, \Omega}{R} \,.\, \cos \vartheta - MR \cos \vartheta \,.\, \sin \vartheta \, \Omega^2 - Mf_{11} = M \frac{d^2 \vartheta}{dt^2} \,. \tag{3}$$

It is usual to write $\frac{d\psi}{dt}$ instead of Ω, ψ being the azimuth at any instant of the plane passing through the axis of the cyclone and the axis of the Earth.

The two following equations express therefore the whole motion of the cyclone.

$$R^2 \cos^2 \vartheta \frac{d\psi}{dt} + k^2 \omega \sin \vartheta = K - \int_o^t l_1 dt \,. \tag{4}$$

$$\frac{k^2 \omega}{R} \,.\, \frac{d\psi}{dt} - R \sin \vartheta \,.\, \left(\frac{d\psi}{dt}\right)^2 - f_{11} \sec \vartheta = \sec \vartheta \frac{d^2 \vartheta}{dt^2} \tag{5}$$

f_{11} of course changes sign with the direction of the azimuthal motion of the cyclone with respect to the Earth's surface. If the cyclone moves to the eastward, $l_1 = l + f_{11} R \cos \vartheta$.

We shall not here discuss whether f_{11} is a constant, or a function of the velocity, or a more complicated function. That probably can be determined by observation. We may write (4)

$$R \cos \vartheta \, v_a + k^2 \omega \sin \vartheta = K - \int_o^t (l - f_{11} R \cos \vartheta) \tag{6}$$

where v_a is the actual azimuthal velocity of a cyclone.

*Since these northward forces act in a plane passing through the axis of the Earth, it is clear that they cannot influence the total moment of momentum about this axis.

In finding the position of the cyclone relatively to the Earth this actual azimuthal velocity will have to be compounded with that of the Earth's surface at the point where the cyclone happens to be. If this compounded velocity happens to be zero, as is usually the case at some point or points of the cyclone's path, this marks a point where the cyclone is changing from a relatively westward to a relatively eastward velocity or vice versa. In the ordinary parabolic paths of tropical hurricanes the point of recurvation is such a point.

The equations (4) and (5) are not in general susceptible of direct integration. But if we consider in Equa. (6) f_{11} to be a constant, and if the cyclone is in a low latitude, so that the term $k^2 \omega \sin \vartheta$ can be neglected without any great error, then we can easily use the equation. The values K and f_{11} will have to be determined by observations. For short periods l_1 might be considered constant, and there would then be only the two values K and l_1 to be determined.

Having determined an approximate value for $\frac{d\psi}{dt}$ at some time ahead, this may be substituted in Equa. (5) and assuming the most probable value for ϑ, the northward velocity $\frac{d\vartheta}{dt}$ might be determined to a certain degree of approximation.

Using an approximate formula $v_p^2 = K(\vartheta - \vartheta_0) - K'(\vartheta - \vartheta_0)^2$ where ϑ_0 is the latitude in which the cyclone originated, and v_p is the polar velocity, the writer has found a close agreement in the cyclones to which it was applied.

We have hitherto supposed that the cyclone was moving in a still and isobaric atmosphere. The chief and vastly preponderating forces which shape the path of the cyclone have already been considered. We shall now consider how far a deviation of the general atmosphere from the normal condition we have supposed can modify the shape of this path.

Let us suppose that the cyclone lies on a gradient between some general High and some general Low. We suppose here merely a gradient without any general motion of the atmosphere (wind) towards the Low. As far as the writer can gather from observations of vortices on the surface of water they are not influenced by ripples passing by them. Their shape is distorted, but unless a wave breaks over them, filling up the funnel, their motion continues as if the ripple had not traversed them. The effect of an existing gradient is merely to cause a slightly greater pressure on one side than the other. Now a cyclone

rarely has the same pressure on all points of its rim. A slightly greater centrifugal force on one side will cause a greater barometrical height at this point and will tend to distort the shape of the cyclone and its stream lines. But no cyclone is a perfect circle. Our equivalent gyroscope is merely the dynamical equivalent of an oval or irregular mass of rotating air. To the extent then that a gradient may modify slightly the shape of a cyclone and therefore its equivalent gyroscope, it may alter the path of a cyclone, but this effect must be extremely slight.

The effect of the prevailing winds, as has been stated before, has been given as the cause of the motion. Now it is true that a cyclone imbedded in a mass of moving air would be carried along with it, but as soon as motion ceased to be parallel, *i. e.* as soon as the plane of the cyclone were turned, gyroscopic forces would be developed. Friction also would divert it from the general surrounding motion.

But a cyclone is a very thin, wide disc of air extending often over hundreds of miles. As it is moving with tremendous force, with its thin edge against the surrounding atmosphere, a wind on one side can have no great effect. True it may crumple up and distort the extreme exposed edge, but in a large cyclone the effect would be inappreciable. Certain it is that cyclones work constantly through the trade wind zones and their paths in general do not seem to be appreciably distorted. In the case of a tornado, which is an extreme form of cyclone, where the diameter may be only a few yards, though the height is considerable, there are no gyroscopic forces set up and its run is extremely limited. Such a whirl is governed simply by the motion of the surrounding air. We see this in a whirl of leaves on a gusty autumn day.

It remains to consider a few of the effects of the motion of a cyclone upon that of the Earth itself. We have seen that the friction couple tends to drive the cyclone backwards around the Earth's axis; but the reaction against this also tends to accelerate the Earth's rotation. Now since these accelerations are always in the same direction for every cyclone, both in the northern and southern hemispheres, the effect must be cumulative. It is possible to compute approximately this effect from the energies and frequencies of cyclones. It must be extremely small. Still as it is cumulative, it might amount to a fraction of a second in the length of the day in a century.

The frictional couple of a cyclone tends to cause the Earth to revolve about an axis directly under its centre, and would do so if the Earth were at rest. The

combination of this tendency and the actual rotation, causes the Earth to revolve about some axis on the meridian of the cyclone and between the actual axis and the axis of the cyclone. Naturally the forces of the friction couple are so small compared with the energy of rotation of the Earth about its axis, that this deviation of the instantaneous axis from the normal axis must be very minute. As cyclones are pretty regularly distributed over the Earth, both in the northern and in the southern hemispheres, their effect in changing the direction of the Earth's axis will, in a long period of time, be apt to be self-eliminatory. The West Indies and the Philippines—the homes of the cyclone—are 180 degrees apart. Still it is possible that they are a factor, even though to a slight extent, in a small (not more than 100 feet) periodical variation which has been observed in latitudes.